Proof of the Derivative of Sine Functions

The proof is given below; your job is to provide the explanation for each step.

Statement	Explanation
f(x) = sin x	Given
$f'(x) = \lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$	
$f'(x) = \lim_{h \to \infty} \frac{\sin(x+h) - \sin(x)}{h}$	
$f'(x) = \lim_{h \to \infty} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	
$f'(x) = \lim_{h \to \infty} \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}$	
$f'(x) = \lim_{h \to \infty} \frac{\sin x (\cos h - 1) + \cos x \sin h}{h}$	
$f'(x) = \lim_{h \to \infty} \left[\frac{\sin x (\cos h - 1)}{h} + \frac{\cos x \sin h}{h} \right]$	
$f'(x) = \lim_{h \to \infty} \left[\frac{\sin x}{1} \bullet \frac{(\cos h - 1)}{h} + \frac{\cos x}{1} \bullet \frac{\sin h}{h} \right]$	
$f'(x) = \sin x(0) + \cos x(1)$	
$f'(x) = \cos x$	

Answer Key

Statement	Explanation
f(x) = sin x	Given
$f'(x) = \lim_{h\to\infty} \frac{f(x+h) - f(x)}{h}$	Definition of a Derivative
$f'(x) = \lim_{h\to\infty} \frac{\sin(x+h) - \sin(x)}{h}$	Substituting sin(x) into the Def'n
$f'(x) = \lim_{h\to\infty} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	Apply formula for sin(A + B)
$f'(x) = \lim_{h\to\infty} \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}$	Reorganize terms
$f'(x) = \lim_{h\to\infty} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$	Factor out sin x
$f'(x) = \lim_{h\to\infty} \left[\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}\right]$	Distribute 'h' as a denominator for both terms.
$f'(x) = \lim_{h\to\infty} \left[\frac{\sin x}{1} \bullet \frac{(\cos h - 1)}{h} + \frac{\cos x}{1} \bullet \frac{\sin h}{h}\right]$	Separate terms into two.
$f'(x) = \sin x(0) + \cos x(1)$	Apply rules from the Squeeze Theorum $\lim_{h\to\infty} \frac{(\cos h - 1)}{h} = 0$ $\lim_{h\to\infty} \frac{\sin h}{h} = 1$
$f'(x) = \cos x$	Simplify Terms